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Optical transmission through generalized Thue-Morse superlattices

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Abstract. In this paper we study the transmission properties of light through generalized Thue-Morse (GTM (m, n)) aperiodic superlattices and obtain the formulae of the transmission coefficients (TCs) analytically at the central wavelength, which are confirmed by numerical simulations. It is found that: (1) all of the 1st generation systems are transparent to the substrates medium B; (2) GTM $(m, 2j)$ systems are transparent to the substrates medium B; (3) GTM $(2i, 2j+1)$ systems are translucent to the substrates medium B except the 1st generation; and (4) when $i \neq j$, transmission through GTM $(2i+1, 2j+1)$ systems attenuates rapidly with the increase of generation number l and sequence parameters m, n. On the other hand, the positional correlations between the constituents of GTM (m, n) aperiodic superlattices responsible for the resonant states are also discussed. Based on the conclusions we study the properties of the amplitude of the electric field vector and find that they are different from those of periodic lattices and chaotic systems.

1. Introduction

People have paid much attention to the quasilattices and quasiperiodic superlattices since the fundamental discovery of quasicrystals [1]. In particular, light propagation through an optical quasiperiodic multilayers has been extensively studied. Kohmoto *et al.* [2, 3] investigated the transmission of light through dielectric multilayers consisting of two kinds of layers, which are arranging following the one-dimensional (1D) Fibonacci quasiperiodic sequence and their theoretical results are confirmed by the dielectric multilayers experiment. Later, the transmission of light through the multilayers arranged by non Fibonaccian sequences was theoretically discussed [4, 5]. Huang and co-workers [6–8] proposed a so-called intergrowth quasiperiodic model, which is an extension of Fibonacci one, and found an interesting switch-like property in the optical transmission coefficient (TC). Based on the studies of Fibonacci model and Intergrowth one, Fu *et al.* [9] constructed the Fibonacci-class ones, which are the perfect extension of Fibonacci one, and the optical transmission through the systems following these sequences was researched by Yang *et al.* [10–12]. The transmission properties of generalized Fibonacci sequences, which are another extensions of Fibonacci model, were explored by Klauzer-Kruszyna *et al.* [13, 14].

On the other hand, being a bridge of linking periodic systems with quasiperiodic models in a geometrical structure, Thue-Morse (TM) systems have attracted considerable attention over the past years. This model was firstly systematically studied by Thue [15] in 1906, and the substitution-generated sequences in the context of topological of dynamics were then researched by Morse [16] in 1921. Cheng *et al.* [17] studied the structure and electronic properties of TM lattices and found that the structure factor is composed of a sequence of δ-function peaks just like quasiperiodic systems. Chattopadhyay and Chakrabarti [18] showed that a TM aperiodic structure presents a unique kind of positional correlation between its constituents, leading to an unattenuated transmission of light as well as electrons through it. Generalized Thue-Morse (GTM (m, n)) models, which are the generally extensions of TM, are interesting aperiodic sequences and have been studied by many groups. Nori *et al.* [19–21] researched the properties of electron, magnetization, and light transmission of TM and GTM (m, n) superlattices in detail. Wang *et al.* [22] studied the properties of trace and antitrace maps for GTM (m, n) models in 2000. The transmission properties of light through the family B of GTM (m, n) multilayers were recently investigated by us [23].

In this paper we study the optical transmission through the generally GTM (m, n) superlattices, not only deduce the total formulae of TCs at the central wavelength, but also find and analyze the interesting transparent, subtransparent, and attenuation characteristics of transmissions. It would be useful for the designing of some optical devices. On the other hand, the positional correlations between the constituents of GTM (m, n) aperiodic superlattices responsible for the resonant states are also discussed. Based on the conclusions we study the properties of the amplitude of the electric field vector and find that they are different from those of periodic lattices and chaotic systems.

We organize this paper as follows: Section 2 is devoted to introduce the substitution rules of GTM (m, n) sequences. In Section 3, we present the analytical results and numerical simulations of the optical transmissions. The positional cor-

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relations between the constituents responsible for the resonant states are discussed in Section 4. Based on Section 4 we study the properties of the amplitude of the electric field vector in Section 5. A brief summary is given in Section 6.

2. Generalized Thue-Morse models

GTM (m, n) models are a class of aperiodic sequences generated by the following substitution rules [21–25]:

$$\begin{aligned} &B \to B^m A^n\,, \\ &A \to A^n B^m\,, \end{aligned} \tag{1}$$

where A^n denotes a string, $AA\ldots A$, of n A's. Starting with a B, the first-two generations of GTM (m, n) are

$$\begin{aligned} &G_1 = B\,, \\ &G_2 = B^m A^n = G_1^m \bar{G}_1^n\,, \end{aligned} \tag{2}$$

where $\bar{G}_1$ is the complementary of G_1, and

$$\begin{aligned} &\bar{G}_1 = A\,, \\ &\bar{G}_2 = A^n B^m = \bar{G}_1^n G_1^m\,. \end{aligned} \tag{3}$$

Equations (1)–(3) show the following recursion relation:

$$\begin{aligned} &G_l = G_{l-1}^m \bar{G}_{l-1}^n\,, \\ &\bar{G}_l = \bar{G}_{l-1}^n G_{l-1}^m\,, \end{aligned} \quad (l \geq 2)\,. \tag{4}$$

3. Characteristics for optical transmission

Propagation of light through GTM (m, n) multilayers is illustrated in Fig. 1, where the (complementary) system is sandwiched by two media of material of type $B(A)$. Using the recursion relations (2)–(4), one can obtain the propagation matrices and TCs through GTM (m, n) systems by means of the propagation theory of the electromagnetic wave as in Ref. [2, 10, 23].

On the other hand, people usually consider the simplest experimental settings and take into account the cases of vertical incidence (*i.e.*, the incident angle $d = d_A = d_B = 0$) and identical optical phase difference for each layer, *i.e.*,

$$n_A d_A = n_B d_B = \frac{\lambda_C}{4}\,, \tag{5}$$

where n_i is the index of refraction of medium i, d_i is the thickness of layer i, and λ_C is the central wavelength. In order to make the quasiperiodicity of the aperiodic GTM (m, n) systems be most effective [2, 10, 23], wavelengths should satisfy the quasi resonance condition $\theta = (l + 1/2)\,p$, where the quasi-phase θ is the optical phase difference between the ends of a layer and can be denoted as follows [2, 10, 23]:

$$\theta_i = \frac{n_i d_i k}{\cos \delta_i}\,, \quad (i = A, B)\,, \tag{6}$$

where k is the wave number in vacuum. Then in this paper we choose

$$\begin{aligned} &\delta = \delta_A = \delta_B = 0\,, \\ &\theta = \theta_A = \theta_B = \pi/2\,. \end{aligned} \tag{7}$$

On this condition, one can obtain the following characteristics for optical transmission through GTM (m, n) systems.

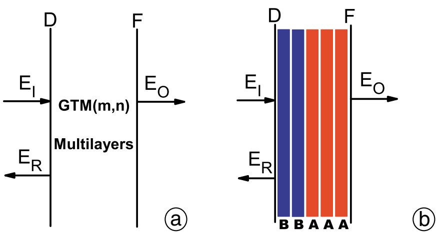

Fig. 1. Propagation of light on the reflective and transmissive surface of GTM (m, n) multilayers, where E_I, E_R and E_O are the input, reflective, and output electric fields, respectively. (**a**) shows the case of general GTM (m, n) systems, and (**b**) shows that of the 2nd generation of GTM (2, 3) system.

3.1 All of the 1st generation systems for GTM (m, n) superlattices are transparent

No matter what m and n are equal to, the TCs of the 1st generation systems for GTM (m, n) superlattices are all equal to 1.0, *i.e.*, all of the 1st generation systems are transparent.

From Eq. (2) one can see that, we choose B as the beginning of GTM (m, n) sequences and then for arbitrary m and n there exist $G_1 = B$. Meanwhile, in this paper we select medium B as the substrates. Then, of course, the 1st generation systems will fuse into one with the substrates without any difference. This makes the 1st generation systems transparent to the substrates.

3.2 All of the GTM $(m, 2j)$ Systems are transparent

The numerical simulations for GTM (1, 2), GTM (3, 2) and GTM (5, 6) systems are illustrated in Fig. 2, where $n_A = 3.0$ and $n_B = 1.2$. The sub-figures in the 1st to 3rd

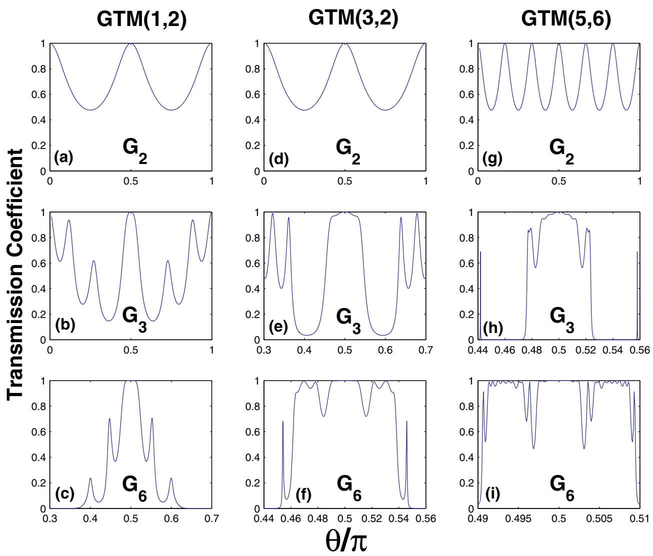

Fig. 2. The relationship between transmission coefficient T and quasi-phase θ/π, where G_i ($i = 2, 3, 6$) denotes the i-th generation, and the 1st to 3rd columns are for the cases of GTM (1, 2), GTM (3, 2) and GTM (5, 6) systems, respectively.

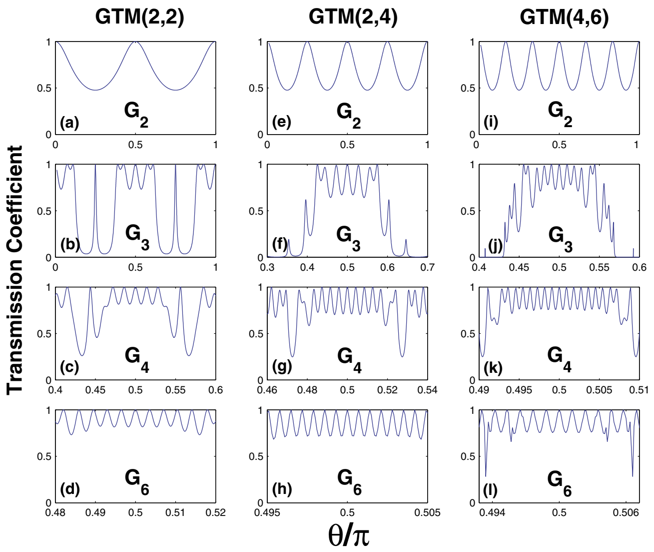

Fig. 3. The relationship between transmission coefficient T and quasi-phase θ/π, where G_i ($i = 2, 3, 4, 6$) denotes the i-th generation, and the 1st to 3rd columns are for the cases of GTM (2, 2), GTM (2, 4) and GTM (4, 6) systems, respectively.

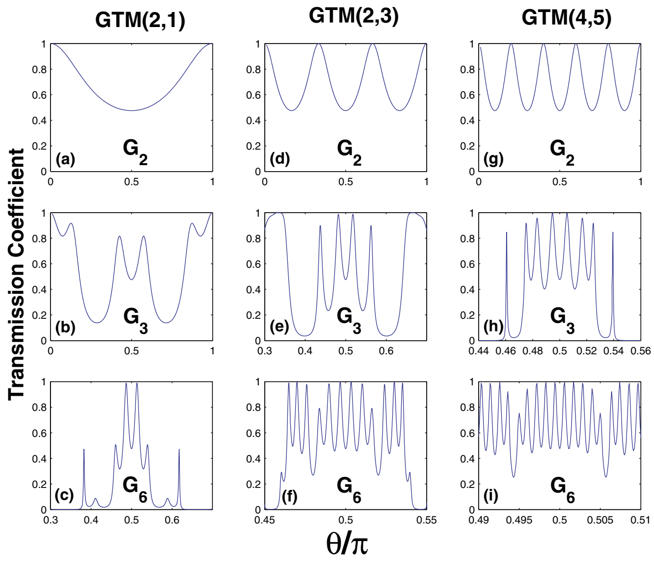

Fig. 4. The relationship between transmission coefficient T and quasi-phase θ/π, where G_i ($i = 2, 3, 6$) denotes the ith generation, and the 1st to 3rd columns are for the cases of GTM (2, 1), GTM (2, 3) and GTM (4, 5) systems, respectively.

rows are for the 2nd, 3rd, and 6th generations, respectively. One can see that, for the three kinds of GTM $(2i+1,\ 2j)$ system, all of the TCs at the points of $\theta/\pi = 0.5$ are equal to 1.0.

The results for GTM (2,2), GTM (2,4) and GTM (4,6) systems are illustrated in Fig. 3, where $n_A = 3.0$ and $n_B = 1.2$. The sub-figures in the 1st to 4th rows are for the 2nd, 3rd, 4th and 6th generations, respectively. It is found that, at the points of $\theta/\pi = 0.5$, all of the TCs are equal to 1.0.

In a word, when n is even, the TCs of GTM (m, n) are all equal to 1.0 at the point of $\theta = \pi/2$, *i.e.*, all of the GTM $(m,\ 2j)$ systems are transparent at the central wavelength.

3.3 GTM $(2i,\ 2j+1)$ Systems are subtransparent except the 1st generation

The numerical simulations for GTM (2, 1), GTM (2, 3) and GTM (4, 5) systems are illustrated in Fig. 4, where $n_A = 3.0$ and $n_B = 1.2$. The sub-figures in the 1st to 3rd rows are for the 2nd, 3rd, and 6th generations, respectively. One can see that, for the three kinds of GTM $(2i,\ 2j+1)$ system, at the points of $\theta/\pi = 0.5$, all of the TCs are equal to 0.4756.

In fact, we deduce analytically that the formula of the TCs for these systems is as follows:

$$T_{G_l}(2i, 2j+1) = \begin{cases} 1.0, & (l = 1) \\ \dfrac{4}{2 + \left(\dfrac{n_B}{n_A}\right)^2 + \left(\dfrac{n_A}{n_B}\right)^2} = T_{\text{sub}}\,, & (l \geq 2)\,. \end{cases} \tag{8}$$

In brief, if m is even and n is odd and from the 2nd generation on, the TCs of GTM (m, n) are all equal to T_{sub} except the first generation. In this paper we set $n_A = 3.0$ and $n_B = 1.2$, then $T_{\text{sub}} \sim 0.4756 \sim {}^1/_2$. That is to say, the GTM $(2i,\ 2j+1)$ systems here are approximately translucent at the central wavelength.

3.4 Transmission through GTM $(2i+1,\ 2j+1)$ systems attenuates rapidly with the increase of parameters m, n and l when $i \neq j$

When m and n are all odd (*i.e.*, $m = 2i+1$ and $n = 2j+1$), it can be demonstrated that the PMs are all diagonal except the 1st generation, and the two non-zero elements for every matrix are count-downs for each other, which decreases or improves rapidly with the increase of both generation number l and the sequence parameters m and n when $m \neq n$. The corresponding TCs can be expressed as follows:

$$T_{G_l}(2i+1, 2j+1) = \begin{cases} 1.0\,, & (l = 1) \\ T_{\text{sub}}\,, & (l = 2)\,, \\ \dfrac{4}{2 + \left(\dfrac{n_B}{n_A}\right)^{2y} + \left(\dfrac{n_A}{n_B}\right)^{2y}} = T_{\text{dec}}\,, & (l \geq 3) \end{cases} \tag{9}$$

where

$$y = |2i - 2j| \cdot (2i + 2j + 2)^{l-3}\,. \tag{10}$$

The numerical simulations for GTM (1, 1), GTM (1, 3) and GTM (5, 7) systems are illustrated in Fig. 5, where $n_A = 3.0$ and $n_B = 1.2$. The sub-figures in the 1st to 4th rows are for the 2nd, 3rd, 4th and 7th generations, respectively. One can see that, for the three kinds of GTM $(2i+1,\ 2j+1)$ system and at the points of $\theta/\pi = 0.5$, (1) the TCs for GTM (1,1) systems equal to 1.0 except the 2nd generation; (2) all of the TCs for the 2nd generations are equal to $0.4756 \sim {}^1/_2$; (3) the TCs for GTM (1, 3) and GTM (5, 7) systems tend rapidly to be zero with the increase of generation; and (4) if $i = j$ (e.g., for GTM (1, 1) system), then $y = 0$ and $T_{\text{dec}} = 1.0$. It

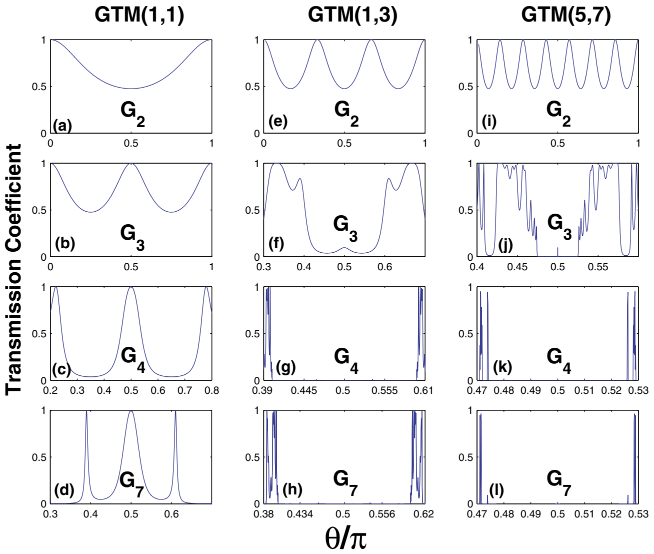

Fig. 5. The relationship between transmission coefficient T and quasiphase θ/π, where G_i ($i = 2, 3, 4, 7$) denotes the i-th generation, and the 1st to 3rd columns are for the cases of GTM (1, 1), GTM (1, 3) and GTM (5, 7) systems, respectively.

means that, from the 3rd generation on, all of the TCs for GTM $(2i+1,\ 2i+1)$ systems are equal to 1.0. The analytic results are confirmed by numerical simulations.

4. Positional correlations between the constituents responsible for the resonant states

In this section, we discuss the positional correlatons between the constituents of GTM (m, n) superlattices which are responsible for the resonant (transparent) states.

4.1 All of the i^{2j} clusters form resonant states

On the condition of Eq. (7), one can demonstrate that the propagation matrix of $2j$ layers of medium i ($i = A, B$) is a unit matrix multiplying $(-1)^j$ and the corresponding TC is equal to 1.0. It means that, after many times of interference every i^{2j} cluster forms a resonant state and becomes a transparent element at the central wavelength. This kind of structures in GTM (m, n) superlattices do not influence the optical transmissivity of the total system and can be decimated from the 1D GTM (m, n) chains, *i.e.*,

$$\alpha\beta^{2j}\gamma \Leftrightarrow \alpha\beta \quad (\alpha, \beta, \gamma = A, B)\ , \tag{11}$$

where $\Leftrightarrow$ means equivalent.

4.2 From the 3rd generation on TM optical superlattices form resonant states

TM model is the simplest one in GTM (m, n) families. Being antisymmetric from the 2nd generation on, TM optical superlattices possess interesting positional correlaton between the constituents. Starting with a B, the 1D TM chain can be obtained based on the recursion Eqs. (2–4) of GTM (m, n) models. It can be demonstrated easily that, from the 3rd generation on TM optical superlattices form resonant states because of the antisymmetry and become transparent at the central wavelength (see Fig. 5). For example, we show the decimating procedure of the 3rd and 4th generations of TM optical superlattices as follows:

$$\begin{aligned} G_3 &= BAAB \Leftrightarrow BB \Leftrightarrow \text{Transparent}\,, \\ G_4 &= BAABABBA \Leftrightarrow BBAA \Leftrightarrow \text{Transparent}\,. \end{aligned} \tag{12}$$

4.3 Positional correlations between constituents of GTM (m, $2j$) families make all of the systems transparet to the substrates

Based on Eq. (11) one can also see that, at the central wavelength the substitution rules for GTM $(m,\ 2j)$ optical multilayers can be simplified as follows:

$$\begin{aligned} B &\to B^m A^{2j} \Leftrightarrow B^m \Leftrightarrow B\,, \\ A &\to A^{2j} B^m \Leftrightarrow B^m \Leftrightarrow B\,. \end{aligned} \tag{13}$$

It means that at the central wavelength, all of the neighboring A clusters in GTM $(m,\ 2j)$ optical superlattices form resonant states and this makes the total systems be equivalent to one layer of medium B. In this paper we choose the substrates medium B, and of course, the positional correlations between constituents of GTM $(m, 2j)$ families make all of the systems transparet to the substrates medium B at the central wavelength (see Figs. 2 and 3).

4.4 Positional correlations between constituents of GTM ($2i$, $2j+1$) families make all of the systems subtransparet to the substrates medium B except the 1st generation

Similarly to Section 4.3 one can also see that, at the central wavelength the substitution rules for GTM $(2i,\ 2j+1)$ optical multilayers can be simplified as follows:

$$\begin{aligned} B &\to B^{2i} A^{2j+1} \Leftrightarrow A\,, \\ A &\to A^{2j+1} B^{2i} \Leftrightarrow A\,. \end{aligned} \tag{14}$$

It means that at the central wavelength, all of the neighboring B clusters in GTM $(2i,\ 2j+1)$ optical superlattices form resonant states and this makes the total systems be equivalent to one layer of medium A except the 1st generation. In this paper we choose the substrates medium B, and of course, the positional correlations between constituents of GTM $(2i,\ 2j+1)$ families make all of the systems subtransparet to the substrates medium B at the central wavelength (see Ref. [2, 10, 23] and Fig. 4).

4.5 Positional correlations between constituents of GTM ($2i+1$, $2j+1$) families make the systems attenuate rapidly with the increase of parameters m, n and l when $m \neq n$

From Eq. (1) one knows that the substitution rules for GTM $(2i+1,\ 2j+1)$ models are as follows:

$$\begin{aligned} B &\to B^{2i+1} A^{2j+1}\,, \\ A &\to A^{2j+1} B^{2i+1}\,, \end{aligned} \quad (i, j \geq 0)\,. \tag{15}$$

Comparing with Secs. 4.3 and 4.4 we notice that, although each $A^{2j}(B^{2i})$ cluster will still form resonant states and can be decimated from the optical superlattices, here every $A(B)$ character will produce a string of $A^{2j+1}B^{2i+1}(B^{2i+1}A^{2j+1})$ in the next generation, which makes optical transmission attenuation. So for the total optical transmission effect, every $A^{2j}(B^{2i})$ cluster can only be decimated from a specific sequence but not from the substitution rules, *i.e.*, the substitution rules can not be simplified for these families. This is the main difference between GTM$(2i+1, 2j+1)$ and GTM$(m, 2j)$ (GTM$(2i, 2j+1)$) systems. Fortunately, during the decimating procedure the simplified result of the l-th generation can be substituted for the item in recursion Eq. (4) directly. The decimating procedure of the GTM$(2i+1, 2j+1)$ systems can be expressed as follows:

$$\begin{cases} G_2 = G_1^{2i+1}\bar{G}_1^{2j+1} = B^{2i+1}A^{2j+1} \Leftrightarrow BA\,, \\ \bar{G}_2 = \bar{G}_1^{2j+1}G_1^{2i+1} = A^{2j+1}B^{2i+1} \Leftrightarrow AB\,, \end{cases} \tag{16}$$

$$\begin{cases} G_3 = G_2^{2i+1}\bar{G}_2^{2j+1} \Leftrightarrow (BA)^{2i+1}(AB)^{2j+1} \Leftrightarrow \begin{cases} i>j: & (BA)^{2i-2j} \\ i=j: & \text{Transparent}\,, \\ i<j: & (AB)^{2j-2i} \end{cases} \\ \bar{G}_3 = \bar{G}_2^{2j+1}G_2^{2i+1} \Leftrightarrow (AB)^{2j+1}(BA)^{2i+1} \Leftrightarrow \begin{cases} i>j: & (BA)^{2i-2j} \\ i=j: & \text{Transparent}\,, \\ i<j: & (AB)^{2j-2i} \end{cases} \end{cases} \tag{17}$$

$$G_4 = \bar{G}_4$$

$$\Leftrightarrow \begin{cases} i>j: & [(BA)^{2i-2j}]^{2i+1}\,[(BA)^{2i-2j}]^{2j+1} \Leftrightarrow (BA)^{(2i-2j)(2i+2j+2)} \\ i=j: & \text{Transparent} \\ i<j: & [(AB)^{2j-2i}]^{2i+1}\,[(AB)^{2j-2i}]^{2j+1} \Leftrightarrow (AB)^{(2j-2i)(2i+2j+2)} \end{cases}, \tag{18}$$

$$G_l = \bar{G}_l \Leftrightarrow (AB)^y, \quad y = |2i-2j| \cdot (2i+2j+2)^{l-3}, \quad (l \geq 3)\,. \tag{19}$$

From Eqs. (16–19) one can deduce the formulae of the TCs of GTM$(2i+1,\ 2j+1)$ optical superlattices at the central wavelength, the result is exactly Eqs. (9, 10). Obviously, the positional correlations between constituents of GTM$(2i+1,\ 2j+1)$ families make the systems attenuate rapidly with the increase of parameters m, n and l when $m \neq n$.

5. Properties of the amplitude of the electric field vector

Generally, the TC through the l-th generation of GTM(m, n) optical superlattices can be defined as follows [2, 10, 23]:

$$T_{G_l} = \frac{|E_O|^2}{|E_I|^2}\,, \tag{20}$$

where E_O and E_I are the amplitudes of the output and input electric fields, respectively (see Fig. 1). In this paper we use the small-signal approximation and plane wave model to investigate the optical transmission of GTM(m, n) systems and do not consider the absorption by thin films. So for periodic lattices (e.g., $(AB)^p$ system) the amplitude of the electric field for the central wavelength at the output interface attenuates with the increase of the periodic number p, the formula can be experessed as follows:

$$T_{\text{periodic}} = \frac{4}{2 + \left(\frac{n_B}{n_A}\right)^{2p} + \left(\frac{n_A}{n_B}\right)^{2p}} = \frac{|E_O|^2}{|E_I|^2}\,. \tag{21}$$

For chaotic systems the transmission result is indeterminate.

Based on Section 4 one can see that for the central wavelength, there are many kinds of GTM(m, n) families forming resonant states and making the amplitude E_O be constant. It is quite different from those of periodic lattices and chaotic systems.

(1) By means of the conclusions of Sections 4.2 and 4.5 one can find that when $m = n$, the GTM(m, m) families form resonant states from the 3rd generation on for the central wavelength and make the amplitude E_O be equal to E_I without any attenuation.

(2) Based on Sections 4.3 and 4.4 one can see that, the GTM$(m, 2j)$ (GTM$(2i, 2j+1)$) families form resonant states and make the total systems be equivalent to one layer of medium $B(A)$ except the 1st generation at the central wavelength. It makes the amplitude E_O be constant (be equal to E_I ($0.707E_I$) to the substrates.

(3) Based on Section 4.5 one can obtain that when $m \neq n$, the GTM$(2i+1, 2j+1)$ families do not form resonant states and make the amplitude E_O attenuate rapidly with the increase of parameters m, n and l when $m \neq n$. It is similar to the property of periodic systems.

6. Brief summary

Firstly, we introduce the substitution rules of GTM(m, n) sequences. By means of the electromagnetic wave theory we then study the propagations of light through GTM(m, n) aperiodic superlattices and obtain the formulae of the TCs analytically.

Taking into account the odevity of the two parameters m and n, we present the formulae in four cases. (1) GTM(m, n) Systems for the 1st generation are all transparent to the substrates medium B, (2) GTM$(m, 2j)$ Systems are all transparent to the substrates medium B, (3) GTM$(2i, 2j+1)$ systems are translucenct, and (4) when $i \neq j$ transmission through GTM$(2i+1, 2j+1)$ systems attenuates rapidly with the increase of m, n and l. The stable transparent property, stable translucency one, and rapidly zero tendency of GTM(m, n) superlattices could be applied to the design of some optical devices. These analytic results are all confirmed by numerical simulations.

The positional correlations between the constituents of GTM(m, n) aperiodic superlattices responsible for the resonant states are also discussed. Based on the conclusions we study the properties of the amplitude of the electric field vector and find that they are different from those of periodic lattices and chaotic systems.

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